Journal of Applied Mechanics and Technical Physics, Vol. 46, No. 5, pp. 652–657, 2005

FORMATION AND AMPLIFICATION OF SHOCK WAVES IN A BUBBLE "CORD"

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UDC 532.593+532.529+534.222

The structure and dynamics of the wave field generated by a bubble system in the form of an axial bubble cylinder (cord) excited by a plane shock wave propagating along the axis in an axisymmetric shock tube are numerically examined. It is shown that consecutive excitation of oscillations of the bubble zone results in formation of a quasi-steady shock wave in the cord and in the ambient liquid. Results of the numerical analysis of the maximum amplitude of the resulting wave as a function of problems parameters are described.

Key words: *bubble media, shock waves, shock tubes.*

Introduction. The problem of creation of explosive hydroacoustic sources [hydroacoustic analogs of laser systems — Shock Amplification by Systems with Energy Release (SASER)] involves the study of wave processes in media capable of accepting the energy being "pumped in." Such media absorb a comparatively weak external pulsed load and re-emit the latter with a significant increase in amplitude (owing to cumulation effects) and possible concentration of energy in a given direction [1].

Among various problem formulations in this field, let us note the experimental and numerical research of explosions of cord high explosive (HE) charges and non-one-dimensional flows with axial symmetry [1–6] in free bubble media. In particular, the results for a number of problems of an underwater explosion with a detailed analysis of the structure and parameters of the wave field of cord and spiral charges, specific features of transformation of shock waves in bubble media, their amplification due to collisions and focusing, and also the problems of formation of bubble detonation waves in chemically active systems are described in [1]. The special features of the evolution of bubble detonation waves in a cylindrical bubble zone located in a volume of a "pure" liquid were examined in [2]. It was shown that origination and propagation of a detonation soliton requires that the radius of the bubble zone should be greater than a certain critical value depending on the caloric content of the gas in the bubbles, the volume content of the gas, and the bubble radius. The wave structure of the reaction zone and detonation velocity in a column of a chemically active bubble medium were numerically analyzed in [3]. It was found that the bubble detonation wave in such a system can propagate with a velocity higher than that in a one-dimensional case.

Among papers dealing primarily with the behavior of various chemically active or passive media capable of radiation generation, we should note the numerical study of Kedrinskii et al. [4] who considered the excitation of a spherical cloud (cluster) of bubbles by a plane shock wave and formation in the cluster of a shock wave with a curved front with a sharp pressure gradient along this front within the framework of the Iordanskii–Kogarko–van Wijngaarden (IKW) model [7–9]. It was shown that, by changing the volume concentration of the gas phase, one can control the coordinate of the wave focusing spot and, in particular, form this spot in the vicinity of the bubble cluster–liquid interface. The amplitude of the wave emitted by the cluster into the liquid can be higher than the amplitude of the wave exciting the cluster by one or two orders. A certain analog of experimental modeling of

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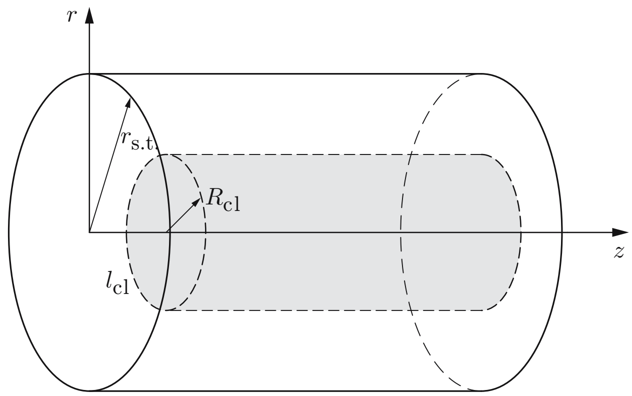

Fig. 1. Layout of a hydrodynamic shock tube with a bubble zone in the form of a co-axial cylinder (bubble cord).

the problem in this formulation was studied in [5], where a polyurethane-foam sphere saturated by a gas was used as a cluster. By means of numerical simulation of shock-wave interaction with a toroidal bubble cluster placed onto the centerline of a hydrodynamic shock tube, it was shown [6] that generation of a toroidal wave by a bubble cluster leads to emergence of directed emission toward the liquid. Another example of wave focusing in axisymmetric geometry was considered in [10], where the problem of a hydrodynamic shock tube with abrupt changes in the cross-sectional size and filled by a chemically active bubble mixture was numerically studied.

The present paper describes a numerical study of generation of acoustic radiation by a passive bubble cord excited by a shock wave propagating in the ambient liquid along this cord.

Formulation of the Problem. Governing Equations. A jump in mass velocity is set at the time $t = 0$ over the entire cross section of the end face of a shock tube of radius $r_{\text{s.t.}}$ filled by water. The central region of the tube contains a cylindrical bubble zone (bubble cluster in the form of a cord) of radius R_{cl} ($R_{\text{cl}} < r_{\text{s.t.}}$) with a volume concentration of the gas phase k_0; the generatrix of this zone is parallel to the z axis (Fig. 1). The gas bubbles in the cluster have an identical radius R_0. The cluster is located at a certain distance l_{cl} from the plane $z = 0$. At $t > 0$, a shock wave propagates from left to right along the cluster surface in the ambient liquid and is refracted inward the cluster. Because of the difference in velocities, the shock wave in the liquid moves ahead of the shock wave in the bubble zone, thus, generating a complicated wave structure in the latter, which obviously depends not only on the zone parameters but also on the cord radius.

Interaction of a plane shock wave with a bubble system leads to focusing of the shock wave inside the cord and its amplification; the degree of amplification is determined by the system parameters and the cord radius R_{cl}. The amplified shock wave propagates in the cord along the axis of symmetry and is emitted into the ambient liquid. The numerical study of formation and propagation of a shock wave in a bubble cord was performed within the framework of a modified IKW model [4], which includes the laws of conservation of mass and momentum for the mean values of pressure p, density ρ, and velocity $\boldsymbol{u}$, and also the equations of state for the liquid component and the bubble medium:

$$\frac{\partial \rho}{\partial t} + \operatorname{div}(\rho \boldsymbol{u}) = 0, \qquad \frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u}(\nabla \boldsymbol{u}) = -\frac{1}{\rho}\nabla p,$$

$$p = p(\rho) = 1 + \frac{\rho_0 c_0^2}{n p_0}\Big[\Big(\frac{\rho}{1-k}\Big)^n - 1\Big], \qquad k = \frac{k_0}{1-k_0}\,\rho\beta^3. \tag{1}$$

Here ρ_0 is the density of the liquid, c_0 is the velocity of sound in the liquid, and k_0 is the initial volume concentration of the gas phase. In the equation of state for the liquid phase, we have $n = 7.15$. System (1) is not closed: the equations of state contain a new variable k, which is the volume concentration of the gas phase in the cluster, containing a dynamic variable $\beta = R/R_0$ (dimensionless bubble radius).

The specific feature of the IKW model is the fact that a physically heterogeneous medium is considered as a homogeneous medium with specific properties of state described by the modified Rayleigh equation

$$\frac{\partial S}{\partial t} = -\frac{3}{2\beta} S^2 - \frac{C_1}{\beta^2} - C_2 \frac{S}{\beta^2} - \frac{p}{\beta} + \beta^{-3\gamma-1}, \tag{2}$$

where

$$S = \frac{\partial \beta}{\partial t}, \qquad C_1 = \frac{2\sigma}{R_0 p_0}, \qquad C_2 = \frac{4\mu}{R_0\sqrt{p_0\rho_0}}.$$

Here σ is the coefficient of surface tension, μ is the viscosity coefficient, and p_0, ρ_0, R_0, $\sqrt{p_0/\rho_0}$, and $R_0\sqrt{\rho_0/p_0}$ are constants used to normalize the system of equations.

Beginning from publications of 1960s–1970s (see the references in [1, Chapters 5 and 7]), this model was used to describe the development of cavitation in the case of underwater explosions near the free surface, the strength of a real liquid containing cavitation cores, and the structure of shock waves and rarefaction waves and their interaction in passive [4, 6] and chemically active [10] bubble media in both one-dimensional and two-dimensional formulations. A comparison of shock-wave processes and the dynamics of state and the structure of bubble media calculated within the framework of this model with experimental data showed that the IKW model and its modifications offer an adequate description of both the wave processes in passive and chemically active bubble media and the dynamics of the medium structure.

In cylindrical coordinates, the solution domain has the form of a rectangle $0 \leqslant z \leqslant z_{\max}$, $0 \leqslant r \leqslant r_{\text{s.t.}}$. Setting the axial component of velocity in the plane $z = 0$ under an assumption that the radial component equals zero defines a steady shock wave with an amplitude P_{sh}. Conditions of symmetry are set at the axis $r = 0$. A condition preventing shock-wave reflection is imposed at the boundary $r = r_{\max}$ (shock-tube wall). Wave exit from the domain with $z = z_{\max}$ is assumed to occur when the second derivatives of all functions with respect to the z coordinate vanish. To solve the gas-dynamic system (1), we used the explicit scheme with directed differences and the splitting scheme described in [11] and adapted to the present problem. Note, the scheme proposed in [12] can also be used in computations. Equation (2) was solved with the use of the fourth-order implicit Runge–Kutta–Merson scheme.

Results of Numerical Computations. We computed the process of formation of the wave field in the bubble cord and ambient liquid in a shock tube of radius $r_{\text{s.t.}} = 15$ cm for a computational domain bounded by the coordinate $z_{\text{s.t.}} = 70$ cm. The cord parameters (volume concentration of the gas phase k_0, bubble radius R_0, and bubble-cord radius R_{cl}) were varied as $k_0 = 0.005$–0.05, $R_0 = 0.01$–0.4 cm, and $R_{\text{cl}} = 0.5$–5 cm; the amplitude of the incident shock wave was varied in the range $P_{\text{sh}} = 1$–12 MPa. Figure 2 shows the typical profiles of the wave field (a), the system of isobars revealing the special features of the wave-field structure as a whole (b) and in the vicinity of the first wave (c), and the qualitative pattern of the distribution of the dimensionless bubble radius in the cord (d). In the latter picture (Fig. 2d), the light domain within the cord ($[-1, +1]$) corresponds to the maximum compression of the bubbles. It is seen that intense pressure oscillations are generated in the medium in the zone of bubble collapse; as a result, a solitary pressure pulse (Fig. 2a) with a fine structure of the shock-wave core (Fig. 2c) is formed in the medium as a whole; a rarefaction phase arises behind this pressure pulse, and the next oscillation starts originating (Fig. 2a). This secondary wave is noticeably weaker.

The computations show (Fig. 2c) that a focusing zone with a curved shock-wave front arises inside the cord; this zone exists even in a thin bubble cord and correlates with a concave zone of the minimum bubble radii. The pressure distribution at the z axis for four different times (Fig. 3) gives an idea about the formation of the wave field in the cord. It is seen that the pressure-distribution dynamics drives the process to a certain asymptotic value: a quasi-steady pressure pulse is formed on the background of the incident wave (see Fig. 3, the plateau with an amplitude of 3 MPa ahead of the front).

An analysis of the influence of the specific fraction of the gas phase in the bubble cord shows that the amplitude of the resulting wave at the z axis increases with increasing k_0 (Fig. 4) and can be approximated by the simple dependence

$$p_{\text{cl}} \approx 250 k_0^{0.6}$$

obtained with no changes in other parameters ($p_{\text{sh}} = 3$ MPa, $R_0 = 0.2$ cm, and $R_{\text{cl}} = 1$ cm) used in the computations plotted in Figs. 2 and 3. Note that p_{cl} defines the value of the maximum amplitude of the quasi-steady wave under conditions close to asymptotic ones on the part of the cord 70 cm long. We can definitely argue that the steady state on this part of the cord is reached only for incident wave amplitudes $p_{\text{sh}} = 0.5$–1.5 MPa.

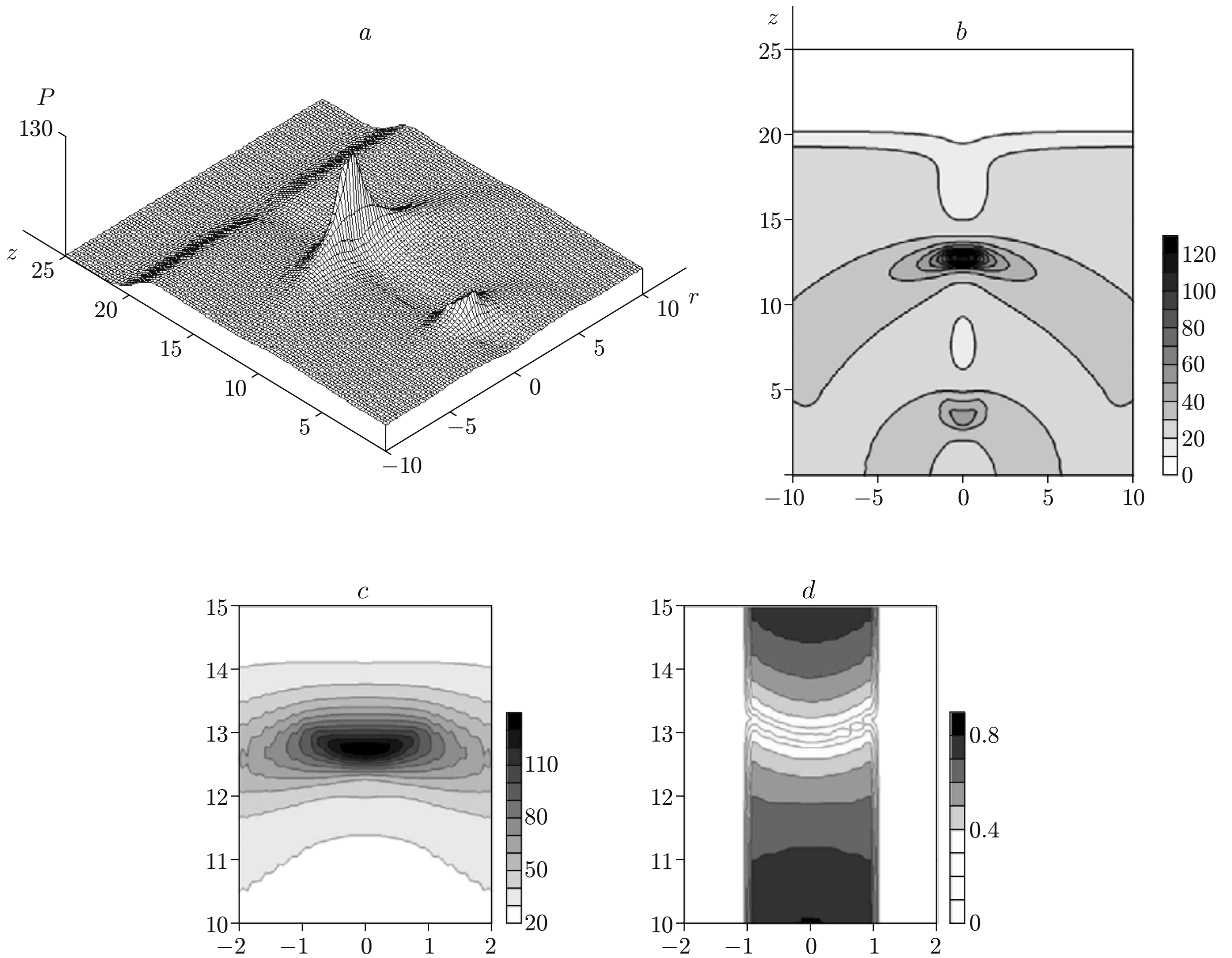

Fig. 2. Typical structure of the wave field (a and b), fine structure of the pressure field in the vicinity of the maximum amplitude of the shock wave (c) and in the vicinity of the bubble zone (d): $k_0 = 0.01$, $R_0 = 0.2$ cm, $P_{\rm sh} = 3$ MPa, and $R_{\rm cl} = 1$ cm.

In studying the wave field generated in a spherical bubble cluster excited by an external plane shock wave, it was shown [4] that the pressure in the focusing spot significantly depends on the number density of bubbles in the cluster with an unchanged volume concentration k_0. According to the results of the wave-field analysis, this effect is also retained in the bubble cord. Figure 5 shows this dependence for three values of the cord radius: 1, 2, and 3 cm. The decrease in amplitude with increasing bubble radius has an exponential character, and wave focusing in the bubble zone noticeably amplifies the resulting wave as the cord radius increases. The order of this dependence can be approximated by the exponential function

$$p_{\rm cl} \approx R_{\rm cl}^{0.65}(11 + 21\,{\rm e}^{-8R_0}),$$

where R_0 and $R_{\rm cl}$ are measured in cm and $p_{\rm cl}$ is measured in MPa.

As was already noted, an increase in the cord radius should facilitate focusing of the shock wave in the vicinity of the axis of symmetry, where the difference in velocities of propagation of the incident shock wave in the liquid surrounding the cord and in the bubble zone is more pronounced. As is seen in Fig. 6, a noticeable increase in amplitude of the resulting wave is observed.

It turned out that the thinner the cord, the faster the formation of the quasi-steady shock wave in the bubble cord. One can again use a simple approximation, which yields results close to the computed points:

$$p_{\rm cl} \approx 1.8 + 14.5R_{\rm cl} - 0.8R_{\rm cl}^2.$$

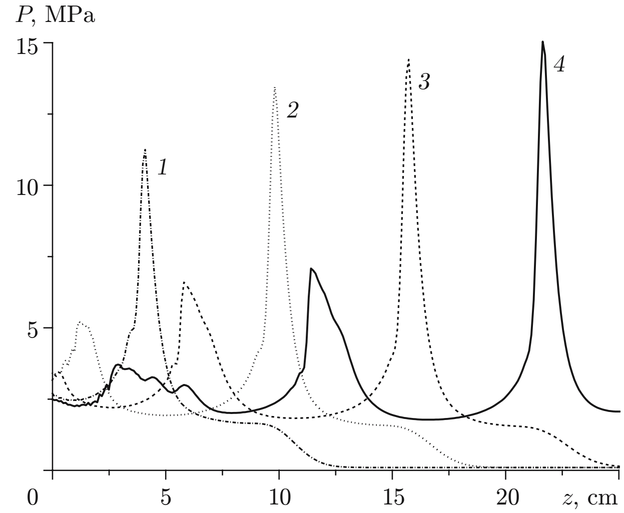

Fig. 3

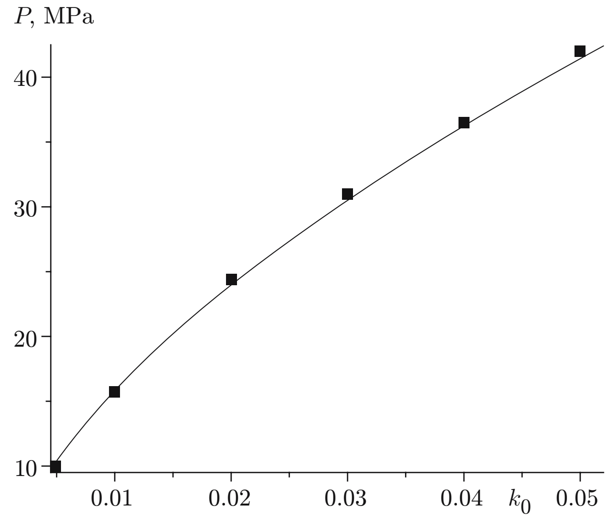

Fig. 4

Fig. 3. Pressure distribution at the axis of the bubble cord for the times of 140 (1), 180 (2), 220 (3), and 260 μsec (4).

Fig. 4. Amplitude of a quasi-steady shock wave versus the volume concentration of the gas phase: the points show the computed data, and the curve approximates these results.

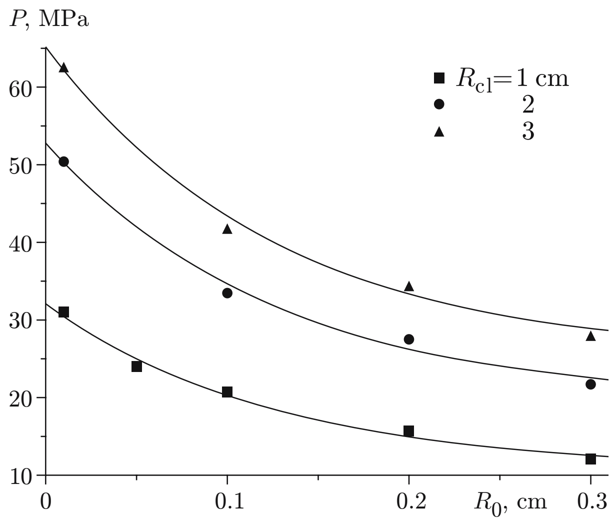

Fig. 5

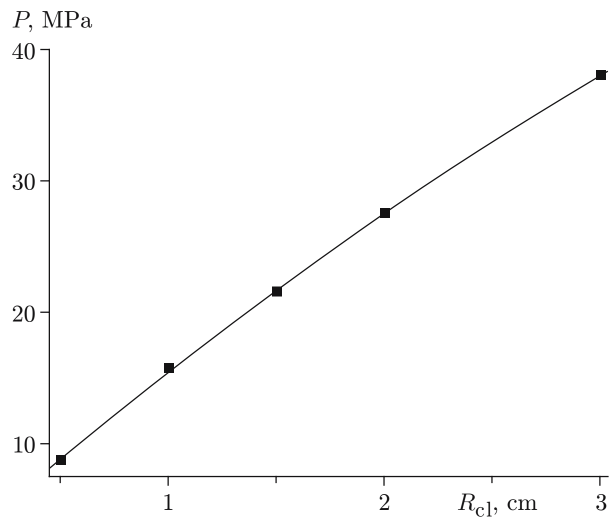

Fig. 6

Fig. 5. Effect of the number density of bubbles in the cord on the amplitude of the resulting wave ($k_0 = 0.01$ and $P_{\rm sh} = 3$ MPa): the points show the computed data, and the curves approximate these results.

Fig. 6. Amplitude of the quasi-steady wave as a function of the bubble-cord radius: the points show the computed data, and the curve approximates these results.

Conclusions. By means of a numerical analysis of the structure and dynamics of the wave field generated in a hydrodynamic shock tube by a bubble system in the form of an axial bubble cylinder (cord), it is shown that a quasi-steady shock wave is formed in the cord and in the ambient liquid. The maximum amplitude of the resulting wave, which is defined by problem parameters, increases with increasing cord radius and volume concentration of the gas phase. For a fixed value of k_0, the amplitude substantially increases with increasing number density of bubbles (decreasing R_0) in a unit volume of the mixture. Approximation dependences that offer adequate estimates of the degree of amplification of the wave field in the bubble cord are derived.

This work was supported by the Foundation "Leading Scientific Schools of Russia" (Grant No. NSh-2073.2003.1) and by the Integration Project No. 22 of the Siberian Division of the Russian Academy of Sciences.

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